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Effect of Phasor Measurement Unit (PMU) on the Network Estimated Variables

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***Abstract* - The classical method of power measurement of a system are iterative and bulky in nature. The new technique of measurement for bus voltage, bus current and power flow is a Phasor Measurement Unit. The classical technique and PMUs are combined with full weighted least square state estimator method of measurement will improves the accuracy of the measurement. In this paper, the method of combining Full weighted least square state estimation method and classical method incorporation with PMU for measurement of power will be investigated. Some cases are tested in view of accuracy and reliability by introducing of PMUs and their effect on variables like power flows are illustrated. The comparison of power obtained on each bus of IEEE 9 and IEEE 14 bus system will be discussed.**

***Keywords* – classical method; phasor measurement units (PMUs); state estimation; full weighted least square (WLS) state.**

I. Introduction

The nature of electrical waves on grid will reflect the health of the system. The electrical wave is consisting of a complex number that represents both the magnitude and phase angle of the sine waves. The "Synchrophasor" is a device which measure nature of waves with respect to time [5] [10]. The PMU is a device that allows the measurement of voltage with multiple current options at each bus. The recently developed PMU [7] [8] [9] will help in deciding to stall power improvement devices at proper location for:-

- Comprehensive planning
- More accurate
- Congestion tracking,
- Advanced warning systems,
- Information sharing,
- Enhancement of System Integrity Protection Schemes (SIPS),
- Quick restoration of Grids,
- Effective grid operation,
- Dynamically manage the grid.

The classical method of measurement of power flow and voltage on a bus of power system are iterative and bulky in nature. The full weighted least square state technique is a nonlinear in nature, but with first order Taylor series it become a linear. Formerly conducted research will help in formulation of a relation between full weighted least square state technique and classical technique. The natural approaches of parameter measurement will extravagance the PMU as computational burden on measurement. The optimal placement of PMU devices is a difficult task for power system state estimation is investigated, in the literature. This paper will reflect the effect of PMU on the accuracy of measurement on state estimation parameters. In case 1, the classical state estimation method without using any PMU. But in case P, the measurement of state estimation with PMU only is discussed.

II. Wls State Estimation Method

Consider the set of measurements given by the vector z:-

$$z = h(x) + e \tag{1}$$

Where:

$$h^T = [h_1(x), h_2(x), h_3(x), \ldots\ldots h_m(x)] \tag{2}$$

$h_i(x)$ is the non-linear function relating measurement i to the state vector x

$x^T = [x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n]$ is the state vector of system

$e^T = [e_1\ e_2\ e_3, \ldots\ldots\ldots e_m]$ is the error measurement in state vector.

The full weighted least square estimator [1] [3] [6] will minimize the following objective function:

$$J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{R_{ii}} = [z - h(x)]^T R^{-1} [z - h(x)] \tag{3}$$

At minimum value of objective function, the first-order optimum condition to be satisfied. It can be expressed as follows:

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x) R^{-1} [z - h(x)] = 0 \tag{4}$$

The Taylor series of non-linear function g(x) can be expanded for the state vector x^k by neglecting the higher order terms [2] will be as

$$g(x) = g(x^k) + G(x^k)(x - x^k) + \ldots\ldots = 0 \tag{5}$$

The Gauss-Newton method is used to solve the above equation:

$$x^{k+1} = x^k - [G(x^k)]^{-1} \cdot g(x^k) \tag{6}$$

where, k is the iteration index, x^k is the state vector at iteration k and G(x) is called the gain matrix and expressed as:

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$$G(x) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k) R^{-1} H(x^k) \quad (7)$$

$$g(x^k) = -H^T(x^k) R^{-1}[z - h(x^k)] \quad (8)$$

Normally, the gain matrix is sparse matrix and decomposed into triangular factors. At each iteration k, the gain matrix are solved by using forward / backward substitutions, where $\Delta x^{k+1} = x^{k+1} - x^k$ and

$$[G(x^k)]\,\Delta x^{k+1} = H^T(x^k) R^{-1}[z - h(x^k)] = H^T(x^k).R^{-1}\Delta z^k \quad (9)$$

These iterations are going on till the maximum variable difference satisfies the condition, ' Max $\left|\Delta x^k\right| < \varepsilon$ '.

III. Classical Method

The traditional/classical method of measurement is consider current as a relation of power flow with respect to bus voltages as

$$I_{ij} = \sqrt{(g_{ij}^2 + b_{ij}^2)(V_i^2 + V_j^2 - 2V_i V_j Cos\theta_{ij})} = \frac{\sqrt{P_{ij}^2 + Q_{ij}^2}}{V_i} \quad (10)$$

The power injection at bus i can be expressed as,

$$S_i = P_i + jQ_i \quad (11)$$

$$P_i = \left|V_i\right| \sum_{j=1}^{N} \left|V_j\right| (G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) \quad (12)$$

$$Q_i = \left|V_i\right| \sum_{j=1}^{N} \left|V_j\right| (G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) \quad (13)$$

The power flow from bus i to bus j are,

$$S_{ij} = P_{ij} + jQ_{ij} \quad (14)$$

$$P_{ij} = \left|V_i\right|^2 (g_{si} + g_{ij}) - \left|V_i\right|\left|V_j\right| (g_{ij}\cos\theta_{ij} + b_{ij}\sin\theta_{ij}) \quad (15)$$

$$Q_{ij} = -\left|V_i\right|^2 (b_{si} + b_{ij}) - \left|V_i\right|\left|V_j\right| (g_{ij}\sin\theta_{ij} - b_{ij}\cos\theta_{ij}) \quad (16)$$

So the Jacobian H matrix will be as

$$H = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial V} \\ \frac{\partial I_{mag}}{\partial \theta} & \frac{\partial I_{mag}}{\partial V} \\ 0 & \frac{\partial V_{mag}}{\partial V} \end{bmatrix} \quad (17)$$

IV. Classical Method With Wls

A PMU device will measure one voltage with multiple current phasors on a bus. Figure 1 shows a 4-bus system example which has single PMU at bus 1. It consist of one voltage with three current phasor measurements, namely as $V_1 \angle \theta_1$, $I_1 \angle \delta_1$, $I_2 \angle \delta_2$ and $I_3 \angle \delta_3$

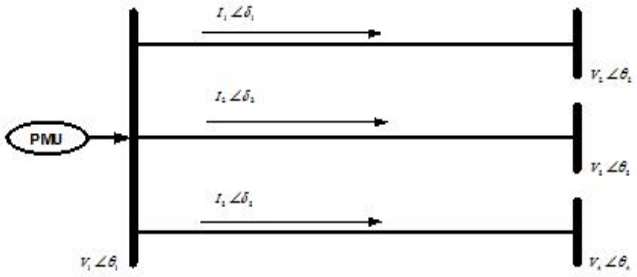

Figure 1. Single PMU Measurement Model

The transmission line normally defined as pie network due to their advantages on system constraints. If y is defined as the series admittance and y_{shunt} as shunt admittance then the current measurements can be in rectangular coordinates as in Fig 2.

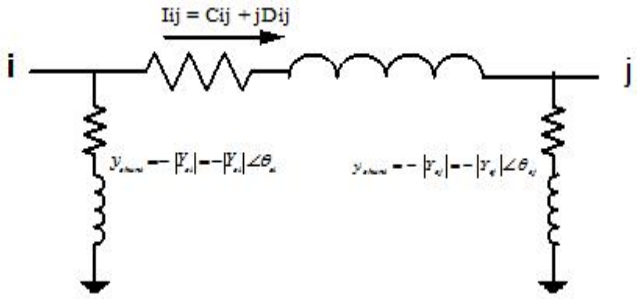

Figure 2. Transmission Line in 'π' Model

The expressions for current in real and imaginary component are as:-

$$C_{ij} = \left|V_i Y_{si}\right|\cos(\delta_i + \theta_{si}) + \left|V_j Y_{ij}\right|\cos(\delta_j + \theta_{ij}) - \left|V_i Y_{ij}\right|\cos(\delta_i + \theta_{ij}) \quad (18)$$

$$D_{ij} = \left|V_i Y_{si}\right|\sin(\delta_i + \theta_{si}) + \left|V_j Y_{ij}\right|\sin(\delta_j + \theta_{ij}) - \left|V_i Y_{ij}\right|\sin(\delta_i + \theta_{ij}) \quad (19)$$

where, state vector is specified as:

$$x = [\left|V_1\right|\angle 0^0, \left|V_2\right|\angle\delta_2, \left|V_3\right|\angle\delta_3 ..... \left|V_N\right|\angle\delta_N]^T \quad (20)$$

The Jacobian H matrix corresponding to their real and reactive parts is:

$$\frac{\partial C_{ij}}{\partial V_i} = \left|Y_{si}\right|\cos(\delta_i + \theta_{si}) - \left|Y_{ij}\right|\cos(\delta_i + \theta_{ij}) \quad (21)$$

$$\frac{\partial C_{ij}}{\partial V_j} = \left|Y_{ij}\right|\cos(\delta_j + \theta_{ij}) \quad (22)$$

$$\frac{\partial C_{ij}}{\partial \delta_i} = -\left|V_i Y_{si}\right|\sin(\delta_i + \theta_{si}) + \left|V_i Y_{ij}\right|\sin(\delta_i + \theta_{ij}) \quad (23)$$

$$\frac{\partial C_{ij}}{\partial \delta_j} = -\left|V_j Y_{ij}\right|\sin(\delta_j + \theta_{ij}) \quad (24)$$

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$$\frac{\partial D_{ij}}{\partial V_i} = |Y_{si}|\sin(\delta_i + \theta_{si}) - |Y_{ij}|\sin(\delta_i + \theta_{ij}) \quad (25)$$

$$\frac{\partial D_{ij}}{\partial V_j} = |Y_{ij}|\sin(\delta_j + \theta_{ij}) \quad (26)$$

$$\frac{\partial D_{ij}}{\partial \delta_i} = |V_i Y_{si}|\cos(\delta_i + \theta_{si}) - |V_i Y_{ij}|\cos(\delta_i + \theta_{ij}) \quad (27)$$

$$\frac{\partial D_{ij}}{\partial \delta_j} = |V_j Y_{ij}|\cos(\delta_j + \theta_{ij}) \quad (28)$$

The measurement vector z will be

$$z = [P_{inj}{}^T, Q_{inj}{}^T, P_{flow}{}^T, Q_{flow}{}^T, |V|^T, \delta^T, C_{ij}{}^T, D_{ij}{}^T]^T \quad (29)$$

Generally, measurements attain by PMUs are more accurate as compared to the traditional measurements. So that measurements performed with PMUs are projected to be more precise and accurate as estimated by classical methods.

V. State Estimation With Pmus

The state vector can be expressed as in rectangular coordinates. The voltage measurement ($V = |V|\angle\theta$) can be state as (V = E + jF), and the current measurement can be state as (I = C + jD). Where series admittance of the line as (g_{ij} + jb_{ij}) and shunt admittance of the transmission line as (g_{si} ??+ jb_{si}). The flow of line current I_{ij} can be expressed as:-

$$I_{ij} = [(V_i - V_j)\times(g_{ij} + jb_{ij})] + [V_i \times (g_{si} + jb_{si})]$$
$$= V_i \times [(g_{ij} + jb_{ij}) + (g_{si} + jb_{si})] - V_j \times (g_{ij} + jb_{ij}) \quad (30)$$

The vector z is state as z = h(x) + e, (where x is a system state vector, h(x) is a linear equations, and e is an error vector). In rectangular coordinates:

z = (H_r + jH_m)(E + jF) + e (31)

where, H = H_r + jH_m , x = E + jF and z = A + jB.

A and B are expressed by:

A = H_r × E – H_m × F (32)

B = H_m × E + H_r × F (33)

In matrix form,

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} H_r & -H_m \\ -H_m & H_r \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix} + e \quad (34)$$

Then, the estimated value $\hat{x} = \hat{E} + j\hat{F}$ can be solved as:-

$$\Delta\hat{x} = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z = G^{-1} H^T R^{-1} \Delta z \quad (35)$$

If we define the linear matrix H_{new} as

$H_{new} = \begin{bmatrix} H_r & -H_m \\ -H_m & H_r \end{bmatrix}$, then the eq. (35) can be written as:

$$\hat{x} = \begin{bmatrix} \hat{E} \\ \hat{F} \end{bmatrix} = (H_{new}{}^T R^{-1} H_{new})^{-1} H_{new}{}^T R^{-1} \begin{bmatrix} A \\ B \end{bmatrix} \quad (36)$$

So that equation for $\hat{x}$ can be written in rectangular forms of z vector and H matrix. They are all in real numbers. In respect of the system, the PMU can deliver more precise information about system parameters. Some cases to be performed on classical measurement set with and without PMUs. The different cases simulations and analysis are as shown in Table I with some IEEE bus systems in the next section.

Table I. Different cases in IEEE System

Cases	Measurements
1	Classical with No PMUs
2	Classical with One PMU
P	Only PMUs

VI. Simulation Results

Some cases are tested for analyzing the system variables accuracy with or without PMU, with the help of MATLAB simulink software. The PMU locations in IEEE 9 Bus System and IEEE 14 Bus System at specific Bus Number are as shown in Table II.

Table II. PMU Locations for Each IEEE System

Type of System	PMU locations at Bus
IEEE 9 System	Bus 2,3
IEEE 14 System	Bus 2,3

The circuit diagram will be shown as in Figure 3 for IEEE 9 bus system and Figure 4 for IEEE 14 bus system.

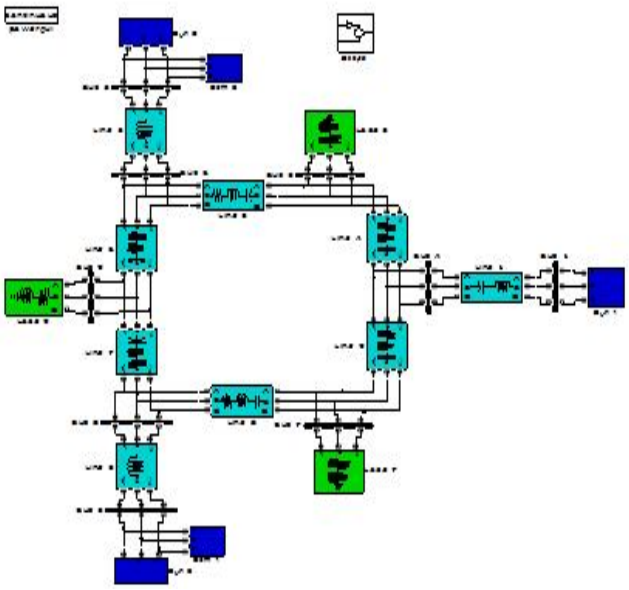

Figure 3. IEEE 9 Bus System

In this segment, IEEE bus systems as IEEE 9 bus system [4] and IEEE 14 bus system [11] are tested with their respective cases to find out the consequences of the PMUs to the precision of the estimated variables.

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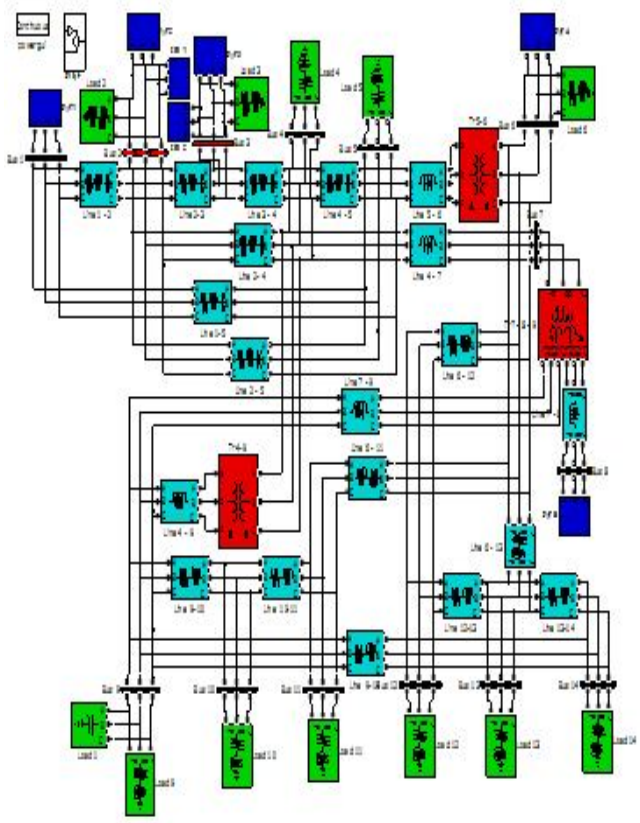

Figure 4. IEEE 14 Bus System

Assume n as the number of variables, m as the number of measurements and ε as the ratio of the number of measurements per the number of variables. During the tests, maintained ε as 1.6. Table III has more detailed information about the measurement numbers for the tests [12].

TABLE III. VARIABLE TYPE AND MEASUREMENTS

Types of System	Var. (n)	Measurements			Ratio ε = (m / n)
		Power	Voltage Magnitude	Total (m)	
IEEE 9 Bus	17	26	1	27	1.6
IEEE 14 Bus	27	42	1	43	1.6

A PMU has much smaller error deviations than classical measurements as 0.0000001.The parameters measured are real power (flow & injected) and reactive power (flow & injected) measurements. The variation of parameters with or without PMU easily reflected in the Figure 5 – 12 as given below:

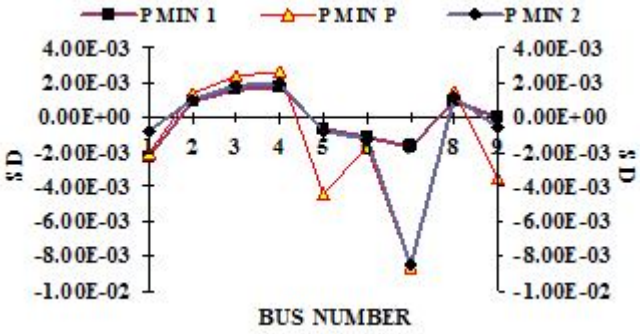

Figure 5. Graph P (SD) vs Bus Number

The graph to be designed on the basis of Standard Deviation (S D) variation in each parameter's on each Bus which is planned for 20 cycle of operation of each IEEE system, it has standard deviation categorized in two categories i.e. Minimum & Maximum variation on actual parameters received at each Bus. The figure 5 & figure 7 will illustrate the variation in Standard Deviation at Minimum & Maximum of Real Power (P) with respect to Each Bus on

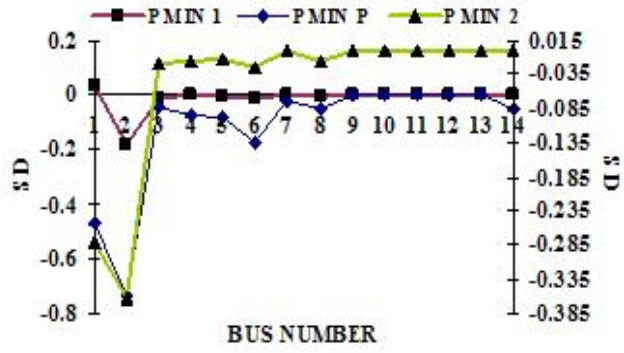

Figure 6. Graph P (SD) vs Bus Number

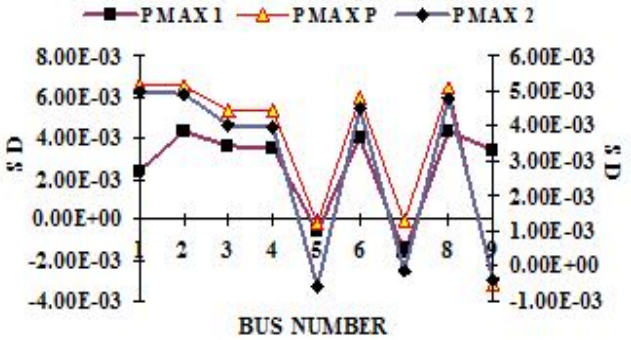

Figure 7. Graph P (SD) vs Bus Number

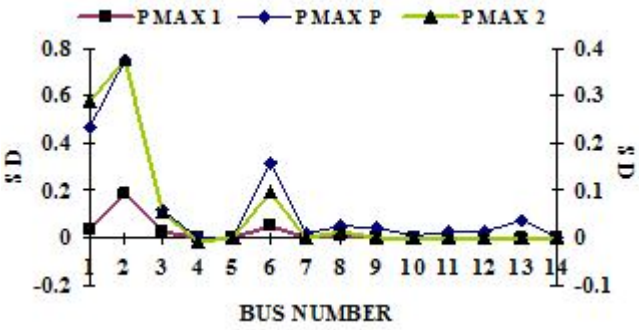

Figure 8. Graph P (SD) vs Bus Number

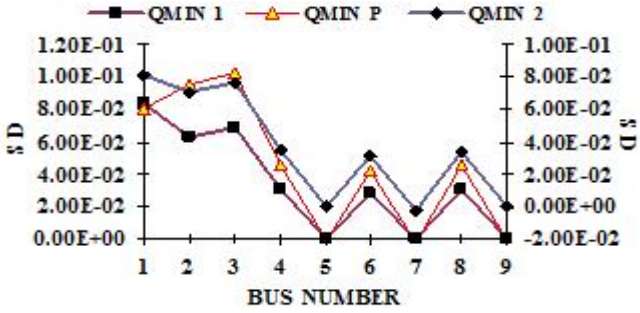

Figure 9. Graph Q (SD) vs Bus Number

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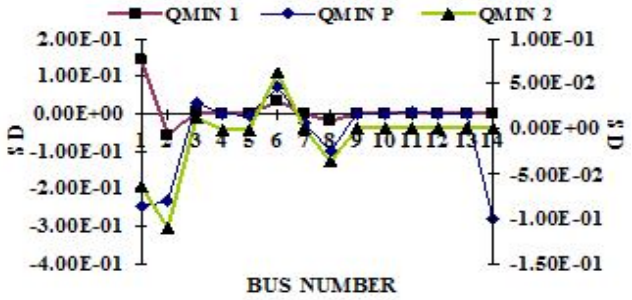

Figure 10. Graph Q (SD) vs Bus Number

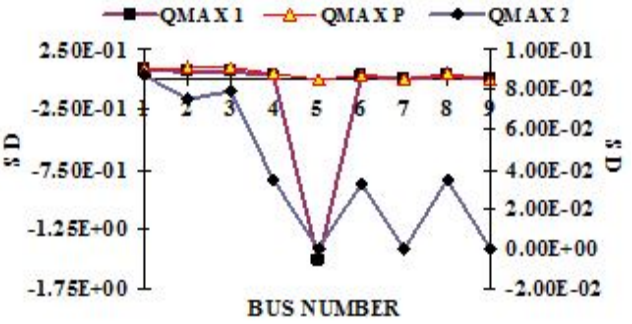

Figure 11. Graph Q (SD) vs Bus Number

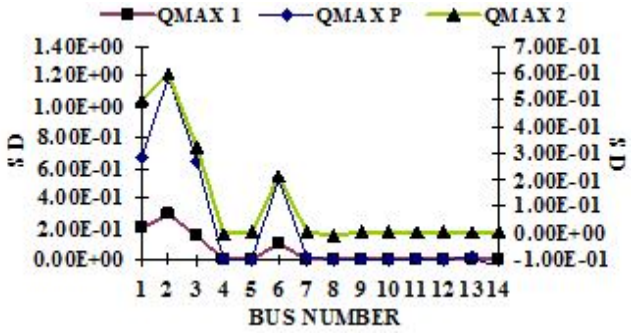

Figure 12. Graph Q (SD) vs Bus Number

varying the number of PMU accretion from 'No PMU (Case 1)' to 'Only PMU (Case P)' for IEEE 9 bus system. Similarly Figure 6 & Figure 8 will illustrate the variation in Standard Deviation at Minimum & Maximum of Real Power (P) with respect to Each Bus on varying the number of PMU accretion from 'No PMU (Case 1)' to 'Only PMU (Case P)' for IEEE 14 bus system. The Standard Deviation in Reactive Power (Q) will be illustrated in Figure 9 – 12 from 'No PMU (Case 1)' to 'Only PMU (Case P)' for IEEE 9 bus system and IEEE 14 bus system.

TABLE IV. AVERAGE S.D. IN ESTIMATED VARIABLES

Case	Value	P		Q	
		9 Bus	14 Bus	9 Bus	14 Bus
1	Min	-2.54E-4	1.16E-5	1.40E-2	9.29E-3
	Max	6.45E-4	1.88E-5	-1.55E-1	1.39E-2
2	Min	-1.08E-3	-3.42E-2	2.90E-2	3.66E-3
	Max	2.22E-3	3.43E-2	3.07E-2	8.04E-2
P	Min	-1.37E-3	-1.02E-1	4.52E-2	-2.88E-2
	Max	3.61E-3	1.16E-1	4.77E-2	2.10E-1

The table IV illustrates that how the Standard Deviation (S D) enhanced at each stage as PMU increase from No PMU to Only PMUs. In IEEE 14 bus system, the S.D. of the estimated Power is approximately 1.88E-05 when there is no PMUs, but at Only PMUs it become 1.16E-01. It means that the S.D. of 'No PMUs' is increased by adding PMUs in system. The interesting thing is that the standard deviation increasing as increasing PMU. Therefore, this result demonstrates that the effectively installing of PMUs will reduce the chances of error in measurement of estimated variables.

CONCLUSION

Now, the classical measurement method with the PMU is able to measure the voltage and current with their magnitude and phasors. The current measurement is implemented but the measurement set as in the rectangular form. The Jacobian matrixes are illustrated in detail for the measurements which include the elements of Equations. The state estimation in the linear formulation is investigated with PMU. All the variables and their respective measurements are improved as all in rectangular form, and then treated separately during the estimation process. Such linear formulation of the PMU data can produce the estimation result by a single calculation without performing the any bulky iteration as in classical methods. If only PMU data measurement set is exist in the real measurement world, and then there will be improvement in the computation time and accuracy estimation as compare to the linear formulation of the state estimation.

The advantages of using PMU will advances the accuracy of the estimated variables. Some cases are tested while gradually increasing the number of PMUs which are added to the measurement set. With the help of advanced accuracy of PMU, it was seen that the estimated accuracy is also increases. One of the motivating thing is that the accuracy improves most effectively when the number of implemented PMUs are around '14 %' of the system buses. It is proved that the quality of the estimation is enhanced by adopting PMU data to the set of measurement. The PMU measurements will provide us improved accuracy and redundancy for the system.

The study is carried out to establish a relationship among classical method, full weighted least square state estimation method and Phasor Measurement Units. Further it is visualized that the information of power variation on each bus of system are more precise and accurate as compared to the classical method. The linear formulation will suggest us a more specific and accurate information of power variation without doing any bulky iteration as performed in the classical methods. The use of PMUs will lead us to enhanced accuracy in results.

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